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B.M.S COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPTEMBER/OCTOBER-2023

M.Sc in Mathematics – 4th Semester

MATHEMATICAL METHODS

Course Code: MM402T **QP Code: 14002**
Duration: 3 Hours **Max Marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. a) Using Laplace transform technique, find the solution of
$$x''(t) = y + \sin t\,, \quad x(0) = 1, x'(0) = 0,$$
$$y''(t) = -x'(t) + \cos t\,, \quad y(0) = -1,\ y'(0) = -1.$$
b) Apply appropriate Fourier transform to solve IBVP: $y_t = y_{xx}\ ,\ 0 \le x \le 1, t \ge 0$ subject to $y(x,0) = f(x),\ 0 \le x \le 1\,,\ y_x(0,t) = y_x(1,t) = 0,\ t \ge 0.$ **(7+7)**

2. a) Find the Hankel transform of $\frac{d^2f}{dx^2} + \frac{1}{x}\frac{df}{dx}$, given $'s'$ is a root of $J_n(sa) = 0,$ where J_n is a Bessel function of order n.
b) Obtain discrete Fourier transform (DFT) of the sequence $f_k = \{1,2,3,4\}$.
c) Distinguish between Fourier transform and wavelet transform. **(6+4+4)**

3. a) Reduce the BVP $y''(x) + \lambda y(x) = 0,\ y(0) = 0,\ y(1) = 0$ into an integral equation.
b) Solve the integral equation $\varphi(x) = x + \int_0^x (\xi - x)\,\varphi(\xi)\ d\xi$ by finding resolvent kernel.
(7+7)

4. a) Determine the eigenvalues and eigen functions of the integral equation
$$\varphi(x) = \lambda \int_0^{2\pi} \cos(x + \xi)\,\varphi(\xi)\,d\xi\ .$$
b) Solve the integral equation $\varphi(x) = x^3 + \int_0^x \sin(x - \xi)\,\varphi(\xi)\,d\xi\,,\ x \in [0, \pi]$. Further evaluate $\varphi(1)$. **(7+7)**

5. a) Find the asymptotic expansion of $I(x) = \int_0^\infty e^{-t^4}\ dt$ as $x \to +\infty$, by the method of integration by parts.
b) Find the leading order behavior of $I(x) = \int_0^\infty e^{-x\sin^4 t}\ dt$ as $x \to +\infty$, by applying Laplace method. **(7+7)**

6. a) State and prove Watson's lemma. Also find the asymptotic behavior of the integral $I(x) = \int_0^5 \frac{e^{-xt}}{1+t^2}\, dt$ as $x \to +\infty$.

   b) Apply method of stationary phase to obtain the leading behavior of the integral $I(x) = \int_0^\infty \cos(xt^2 - t)\, dt$ as $x \to +\infty$. **(7+7)**

7. a) Find the leading two term regular perturbation solution of

$$y'' + \in y' + y = 0\,; \qquad y(0) = 1, y'(0) = 0.$$

   b) Find the first two term periodic solution of $y'' + y + \in y^3 = 0\,; y(0) = a, y'(0) = 0.$ **(7+7)**

8. a) Obtain one term uniformly valid solution of

$$\in y'' + (1 + x)y' + y = 0,\ y(0) = 1, y(1) = 1.$$

   b) Find the WKB solution of $\in^2 y''(x) = (1 + x^2)^2\, y(x)$ with $y(0) = 0, y'(0) = 1.$ **(7+7)**

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